On $\tau_1\tau_2$-$g$-Open Sets in Bitopological Spaces

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Abstract
In this paper, we introduced and studied a new kind of generalized open set called $\tau_1\tau_2$-$g$-open set in a bitopological space $(X, \tau_1, \tau_2)$. The properties of this $\tau_1\tau_2$-$g$-open set are studied and compared with some of the corresponding generalized open sets in general topological spaces and bitopological spaces. We also defined the $\tau_1\tau_2$-$g$-continuous function and studied some its properties.

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1 Introduction

A space $X$ equipped with two arbitrary topologies $\tau_1$ and $\tau_2$ is defined by J. C. Kelley [1] as the bitopological space in 1963 and denoted it by a triple $(X, \tau_1, \tau_2)$ to generalize a topological space $(X, \tau)$. Every bitopological space $(X, \tau_1, \tau_2)$ can be regarded as a topological space $(X, \tau)$ if $\tau_1 = \tau_2 = \tau$. A topological space occurs for every metric spaces but the bitopological spaces occurs for quasi-metric spaces. A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called open if $A$ is both $\tau_1$-open and $\tau_2$-open. In mathematics, and more specifically in topology, an open set is an abstract concept generalizing the idea of an open interval in the real line. The open sets play some role in properties of topological spaces such as once a choice of open sets is made, the properties of continuity, connectedness, and compactness, which use notions of nearness, can be defined using these open sets. The different forms of open sets were studied in past few years. Levine [2] defined that the complement of $g$-closed set is a $g$-open set in 1970. A. Csaszar extended a significant contribution to the theory of generalized open sets recently. There were many different kind of generalized open sets on topological spaces and on bitopological spaces introduced by different authors. As an example, Bhattacharyya and Lahiri [3], Maki, Devi and Balachandran [4] and Keskin and Noiri [5] have introduced and studied $sg$-open sets, $g\alpha$-open sets and $bg$-open sets. In this paper, we introduce another kind of generalized open set in the bitopological space and compare this with some of the corresponding generalized open sets and then analyzed its properties.

2 Preliminaries

Throughout this paper, we represent $X$ and $Y$ as the bitopological spaces $(X, \tau_1, \tau_2)$ and $(Y, \sigma_1, \sigma_2)$ on which no separation axioms are assumed unless otherwise stated. For a subset of $A$ of $X$, $\tau_i$-$\text{cl}(A)$ denotes the closure of $A$ and $\tau_i$-$\text{int}(A)$ denotes the interior of $A$, respectively with respective to the topology $\tau_i$.

In the topological space $(X, \tau)$, we recall the following closed sets.

**Definition 2.1.** A subset $A$ of a topological space $(X, \tau)$ is called a

1. **regular closed** [6] if $A \subseteq \text{cl}(\text{int}(A))$.
2. **$\omega$-closed** [7] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $\tau$.
3. **semi closed** [8] if $\text{int}(\text{cl}(A)) \subseteq A$.
4. **$\alpha$-closed** [9] if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

The complements of the above mentioned closed sets are their respective open sets.

The semi interior (respectively, $\alpha$-interior, semi pre interior, $\delta$-interior and $b\delta$-interior) of a subset $A$ of a space $(X, \tau)$ is the union of all semi open (respectively, $\alpha$-open, semi pre open, $\delta$-open and $b\delta$-open) sets contained in $A$ and is denoted by $\text{sint}(A)$ (respectively, $\alpha\text{int}(A)$, $\text{spint}(A)$, $\text{int}_\delta(A)$ and $\text{b}\delta\text{int}(A)$).

We also recall some generalized closed sets defined in a topological space $(X, \tau)$.

**Definition 2.2.** A subset $A$ of a topological space $(X, \tau)$ is called a

1. **$g$-closed** [2] (generalized closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $\tau$.
2. **$\tau_1 \tau_2$-$g$-closed** [10] if $\tau_i$-$\text{cl}(A) \subseteq U_i$ whenever $A \subseteq U_i$ and $U_i$ is $\tau_i$-open for each $i = 1, 2$.
3. **gs-$g$-closed** [11] (generalized semi closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $\tau$. 

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4. \(r\)-\(g\)-closed [12] (regular generalized closed) if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a regular open in \(\tau\).

5. \(\alpha\)-\(g\)-closed [13] (generalized semi pre closed) if \(acl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(\tau\).

6. \(g\)-\(p\)-closed [14] (generalized semi closed) if \(pcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(\tau\).

7. \(gsp\)-closed [15] (generalized semi pre closed) if \(spcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(\tau\).

8. \(rg\)-\(g\)-closed [16] (regular generalized \(b\) \(g\)-closed) if \(bgcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a regular open in \(\tau\).

9. \(\delta\)-\(g\)-closed [17] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a semi open in \(\tau\).

10. \(\delta\)-\(g\)-closed [18] if \(clf(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a \(\delta\)-open in \(\tau\).

11. \((gsp)^{-}\)-closed [19] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is a \((gsp)^{-}\)-open in \(\tau\).

The complements of the above mentioned generalized closed sets are their respective generalized open sets.

Now we recall some generalized closed sets in a bitopological space \((X, \tau_1, \tau_2)\).

**Definition 2.3.** A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called a

1. \(\tau_1\tau_2\)-\(g\)-closed [20] (\(\tau_1\tau_2\)-generalized closed) if \(\tau_2\)-\(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1\)-open.

2. \(\tau_1\tau_2\)-\(sg\)-closed [21] (\(\tau_1\tau_2\)-semi generalized closed) if \(\tau_2\)-\(scl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1\)-semi open.

3. \(\tau_1\tau_2\)-\(gs\)-closed [22] (\(\tau_1\tau_2\)-generalized semi closed) if \(\tau_2\)-\(scl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1\)-open.

4. \(\tau_1\tau_2\)-\(og\)-closed [23] (\(\tau_1\tau_2\)-\(\alpha\)-generalized closed) if \(\tau_2\)-\(acl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1\)-open.

5. \(\tau_1\tau_2\)-\(oa\)-closed [23] (\(\tau_1\tau_2\)-generalized \(\alpha\)-closed) if \(\tau_2\)-\(ocl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1\)-\(\alpha\)-open.

6. \(\tau_1\tau_2\)-\(\delta\)-closed [22] if \(\tau_2\)-\(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\tau_1\)-\(\delta\)-open.

The complements of the above mentioned generalized closed sets in bitopological spaces are their respective generalized open sets in the corresponding bitopological spaces.

**Definition 2.4.** Let \(\tau_1\) and \(\tau_2\) be two topologies on a set \(X\) such that \(\tau_1\) is contained in \(\tau_2\). Then, the topology \(\tau_1\) is said to be a coarser (weaker or smaller) topology than \(\tau_2\).

## 3 Generalized \(\tau_1\tau_2\)-\(\delta\)-Open Sets

**Definition 3.1.** A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called a \(\tau_1\tau_2\)-\(\delta\)-open if \(F_i \subseteq \tau_i\)-\(int(A)\) whenever \(F_i \subseteq A\) and \(F_i\) is \(\tau_i\)-closed for each \(i = 1, 2\).

**Example 3.2.** Let \(X = \{a, b, c\}\), \(\tau_1 = \{\emptyset, \{a, b\}, X\}\), and \(\tau_2 = \{\emptyset, \{b\}, \{a, c\}, X\}\). Then, \(\emptyset, \{a\}, \{b\}, \{a, b\}\) and \(X\) are the \(\tau_1\tau_2\)-\(\delta\)-open sets in \((X, \tau_1, \tau_2)\).

**Theorem 3.3.** The intersection of two \(\tau_1\tau_2\)-\(\delta\)-open sets is a \(\tau_1\tau_2\)-\(\delta\)-open set.
Proof. Let $A$ and $B$ be two $\tau_1\tau_2\mathring{g}$-open sets. Then, $F_i \subseteq \tau_i \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is $\tau_i$-closed for each $i = 1, 2$ and $F_i \subseteq \tau_i \text{int}(B)$ whenever $F_i \subseteq A$ and $F_i$ is $\tau_i$-closed for each $i = 1, 2$. Then, we have $F_i \subseteq \tau_i \text{int}(A \cap B)$ whenever $F_i \subseteq (A \cap B)$ and $F_i$ is $\tau_i$-closed for each $i = 1, 2$. Therefore, $A \cap B$ is a $\tau_1\tau_2\mathring{g}$-open set.

The union of two $\tau_1\tau_2\mathring{g}$-open sets need not be a $\tau_1\tau_2\mathring{g}$-open set. This can be seen from the following example.

**Example 3.4.** Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, and $\tau_2 = \{\phi, \{b\}, X\}$. If $A = \{a\}$ and $B = \{c\}$, then the sets $A$ and $B$ are $\tau_1\tau_2\mathring{g}$-open; but, $A \cup B = \{a, c\}$ is not a $\tau_1\tau_2\mathring{g}$-open set.

**Theorem 3.5.** Every $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ is a $g$-open set in both $\tau_1$ and $\tau_2$.

Proof. Let $A$ be any $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ and $F_i$ be any closed set in $(X, \tau_i)$ contained in $A$ for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed in $(X, \tau_i)$ for $i = 1, 2$. Since $\tau_i \text{int}(A) \subseteq \tau_i \text{sint}(A)$, $F_i \subseteq \tau_i \text{sint}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed in $(X, \tau_i)$ for $i = 1, 2$. Therefore, $A$ is a $g$-open set in both $\tau_1$ and $\tau_2$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.6.** Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. Then, the set $\{c\}$ is a $g$-open set in both $\tau_1$ and $\tau_2$. But, it is not a $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$.

**Theorem 3.7.** Every $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ is a regular generalized open set or $rg$-open set in both $\tau_1$ and $\tau_2$.

Proof. Let $A$ be any $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ and $F_i$ be any regular closed set in $(X, \tau_i)$ contained in $A$ for $i = 1, 2$ respectively. Since every regular closed set is a closed set, $F_i \subseteq \tau_i \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is regular closed set in $(X, \tau_i)$ for $i = 1, 2$. Therefore, $A$ is a $rg$-open set in both $\tau_1$ and $\tau_2$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.8.** Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b, c\}, \{b\}, X\}$, and $\tau_2 = \{\phi, \{a\}, \{b\}, \{a, c\}, \{a\}, X\}$. Then, the set $\{c\}$ is a regular generalized open set in both $\tau_1$ and $\tau_2$. But, it is not a $\tau_1\tau_2\mathring{g}$-open set.

**Theorem 3.9.** Every $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ is a $og$-open set in both $\tau_1$ and $\tau_2$.

Proof. Let $A$ be any $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ and $F_i$ be any closed set in $(X, \tau_i)$ contained in $A$ for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed in $(X, \tau_i)$ for $i = 1, 2$. Since $\tau_i \text{int}(A) \subseteq \tau_i \text{a int}(A)$, $F_i \subseteq \tau_i \text{a int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed in $(X, \tau_i)$ for $i = 1, 2$. Therefore, $A$ is a $og$-open set in both $\tau_1$ and $\tau_2$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.10.** Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, and $\tau_2 = \{\phi, \{a, c\}, X\}$. Then, the set $\{b, c\}$ is a $og$-open set in both $\tau_1$ and $\tau_2$. But, it is not a $\tau_1\tau_2\mathring{g}$-open set.

**Theorem 3.11.** Every $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ is a $gp$-open set in both $\tau_1$ and $\tau_2$.

Proof. Let $A$ be any $\tau_1\tau_2\mathring{g}$-open set in $(X, \tau_1, \tau_2)$ and $F_i$ be any closed set in $(X, \tau_i)$ contained in $A$ for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed set in $(X, \tau_i)$ for $i = 1, 2$. Since $\tau_i \text{int}(A) \subseteq \tau_i \text{p int}(A)$, $F_i \subseteq \tau_i \text{p int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed set in $(X, \tau_i)$ for $i = 1, 2$. Therefore, $A$ is a $gp$-open set in both $\tau_1$ and $\tau_2$. 

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Let $F$ be closed for $i \delta \tau$. Hence every $F \in \tau$ is regular closed for $i \delta \tau$. But, it is not a $\tau_1 \tau_2 \delta \tau$-open.

**Theorem 3.13.** Every $\tau_1 \tau_2 \delta \tau$-open set in $(X, \tau_1, \tau_2)$ is a $gsp$-open set in both $\tau_1$ and $\tau_2$.

*Proof.* Let $A$ be any $\tau_1 \tau_2 \delta \tau$-open set in $(X, \tau_1, \tau_2)$ and $F_i$ be any closed set in $(X, \tau_i)$ contained in $A$ for $i = 1, 2$ respectively. Then, $F_i \subseteq \tau_i \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed set in $(X, \tau_i)$ for $i = 1, 2$. Since $\tau_i \text{int}(A) \subseteq \tau_i \text{sp} \text{int}(A)$, $F_i \subseteq \tau_i \text{sp} \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is closed set in $(X, \tau_i)$ for $i = 1, 2$. Therefore, $A$ is a $gsp$-open set in both $\tau_1$ and $\tau_2$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.14.** Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{a\}, X\}$. Then, the set $\{b, c\}$ is a $gsp$-open set in both $\tau_1$ and $\tau_2$. But, it is not a $\tau_1 \tau_2 \delta \tau$-open.

**Theorem 3.15.** Every $\omega$-open set in both $\tau_1$ and $\tau_2$ is a $\tau_1 \tau_2 \delta \tau$-open set.

*Proof.* Let $A$ be $\omega$-open set in both $\tau_1$ and $\tau_2$ and $F_i$ be any closed set in $\tau_i$ contained in $A$ for $i = 1, 2$ respectively. Then, $F_i \subseteq \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is a semi closed set in $\tau_i$ for $i = 1, 2$. Since every semi closed set is a closed set, $F_i \subseteq \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is a closed set in $\tau_i$ for $i = 1, 2$. Hence every $\omega$-open set in both $\tau_1$ and $\tau_2$ is a $\tau_1 \tau_2 \delta \tau$-open set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.16.** Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, X\}$; $\tau_2 = \{\phi, \{b\}, \{a\}, X\}$ and $A = \{a, b\}$. Hence the set $A$ is a $\tau_1 \tau_2 \delta \tau$-open. But, it is not a $\omega$-open set in $\tau_1$ and $\tau_2$.

**Theorem 3.17.** Every $\rho \delta$-open set in both $\tau_1$ and $\tau_2$ is a $\tau_1 \tau_2 \delta \tau$-open set.

*Proof.* Let $A$ be $\rho \delta$-open set in both $\tau_1$ and $\tau_2$ and $F_i$ be any closed set in $\tau_i$ contained in $A$ for $i = 1, 2$ respectively. Then, $A = \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is a regular closed in $\tau_i$ for $i = 1, 2$. So, $F_i \subseteq \text{int}(A)$ whenever $F_i \subseteq A$ and $F_i$ is a closed in $\tau_i$ for $i = 1, 2$ as every regular closed set is closed. Hence every $\rho \delta$-open set in both $\tau_1$ and $\tau_2$ is a $\tau_1 \tau_2 \delta \tau$-open set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.18.** Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$; $\tau_2 = \{\phi, \{c\}, \{b, c\}, X\}$ and $A = \{a, c\}$. Hence the set $A$ is a $\tau_1 \tau_2 \delta \tau$-open. But, it is not a $\rho \delta$-open set in $\tau_1$ and $\tau_2$.

**Theorem 3.19.** If $A$ is a $\tau_1 \tau_2 \delta \tau$-open set, then $A$ is $rgb^2$-open set in both $\tau_1$ and $\tau_2$.

*Proof.* Let $A$ be any $\tau_1 \tau_2 \delta \tau$-open set in $X$ such that $F_i \subseteq A$ and $F_i$ is regular closed of $\tau_i$ for $i = 1, 2$ respectively. Hence $A$ is $g$-open set in $(X, \tau_1)$ and $(X, \tau_2)$ as every regular closed set is closed, $F_i$ is closed for $i = 1, 2$. Also, $F_i \subseteq \tau_i \text{int}(A) \subseteq \tau_i \text{sp} \text{int}(A)$ for $i = 1, 2$. Therefore, $F_i \subseteq \tau_i \text{sp} \text{int}(A)$ and $F_i$ is regular closed for $i = 1, 2$, Hence $A$ is $rgb^2$-open set in $\tau_1$ and $\tau_2$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.20.** Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, X\}$; $\tau_2 = \{\phi, \{b\}, \{a\}, X\}$ and $A = \{b, c\}$. Hence the set $A$ is a $rgb^2$-open set in both $\tau_1$ and $\tau_2$. But, it is not a $\tau_1 \tau_2 \delta \tau$-open set.

**Theorem 3.21.** Every $\delta \tau$-open set in both $\tau_1$ and $\tau_2$ is a $\tau_1 \tau_2 \delta \tau$-open set.
Proof. Let \( A \) be an \( \delta g \)-open set in \( \tau_1 \) and \( \tau_2 \) and \( F_i \) is any closed set contained in \( A \) in \((X, \tau_i)\) for \( i = 1, 2 \), respectively. Since every closed set is \( g \)-closed and \( A \) is \( \delta g \)-open set in \( \tau_1 \) and \( \tau_2 \), \( F_i \subseteq \tau_i\text{-int}(A) \) for every subset \( A \) of \( X \) for \( i = 1, 2 \). Since \( F_i \subseteq \tau_i\text{-int}(A) \subseteq \tau_i\text{-int}(A) \), \( F_i \subseteq \tau_i\text{-int}(A) \), and hence \( A \) is \( g \)-open set in \( \tau_1 \) and \( \tau_2 \). Therefore, \( A \) is \( \tau_1\tau_2\text{-}\delta g \)-open set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.22.** Let \( X = \{a, b, c\}; \tau_1 = \{\phi, \{b\}, \{a, c\}, X\}; \tau_2 = \{\phi, \{c\}, \{a, b\}, X\} \) and \( A = \{b, c\} \). Hence the set \( A \) is a \( \tau_1\tau_2\text{-}\delta g \)-open. But, it is not a \( \delta g \)-open set in \( \tau_1 \) and \( \tau_2 \).

**Theorem 3.23.** Every \( \tau_1\tau_2\text{-}\delta g \)-open set is strongly \((gsp)^*\)-open set in both \( \tau_1 \) and \( \tau_2 \).

**Proof.** Let \( A \) be \( \tau_1\tau_2\text{-}\delta g \)-open set. Then, \( F_i \subseteq \tau_i\text{-int}(A) \) whenever \( F_i \subseteq A \) and \( F_i \) is closed in \( \tau_i \) for \( i = 1, 2 \) respectively. Now, let \( F_i \subseteq A \) and \( F_i \) be \((gsp)^*\)-open set in \( \tau_i \) for \( i = 1, 2 \) respectively. Since every closed set is a \((gsp)^*\)-closed set, we have \( F_i \subseteq \tau_i\text{-int}(A) \) whenever \( F_i \subseteq A \) and \( F_i \) is \((gsp)^*\)-closed in \( \tau_i \) for \( i = 1, 2 \). But, \( F_i \subseteq \tau_i\text{-int}(A) \subseteq \tau_i\text{-int(cl}(A)) \) whenever \( F_i \subseteq A \) and \( F_i \) is \((gsp)^*\)-closed in \( \tau_i \) for \( i = 1, 2 \). Hence \( F_i \subseteq \tau_i\text{-int(cl}(A)) \) whenever \( F_i \subseteq A \) and \( F_i \) is \((gsp)^*\)-closed in \((X, \tau_i)\) for \( i = 1, 2 \). Therefore, \( A \) is strongly \((gsp)^*\)-open set in \( \tau_1 \) and \( \tau_2 \).

The converse of the above theorem need not be true as seen from the following example.

**Example 3.24.** Let \( X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, \{a, b\}, X\}; \tau_2 = \{\phi, \{c\}, \{b, c\}, X\} \) and \( A = \{a, c\} \). Hence the set \( A \) is a strongly \((gsp)^*\)-open set in \( \tau_1 \) and \( \tau_2 \). But, it is not a \( \tau_1\tau_2\text{-}\delta g \)-open.

**Theorem 3.25.** If \( \tau_1 \) is coarser than \( \tau_2 \), then every \( \tau_1\tau_2\text{-}\delta g \)-open set is a \( \tau_1\tau_2\text{-}g \)-open set.

**Proof.** Let \( A \) be \( \tau_1\tau_2\text{-}g \)-open set. Then, \( F_i \subseteq \tau_i\text{-int}(A) \) whenever \( F_i \subseteq A \) and \( F_i \) is \( \tau_i \)-closed for each \( i = 1, 2 \). Since \( \tau_1 \) is coarser than \( \tau_2 \), we have \( F \subseteq \tau_2\text{-int}(A) \), whenever \( F \subseteq A \), \( F \) is \( \tau_1 \)-closed. Hence \( A \) is a \( \tau_1\tau_2\text{-}g \)-open set.

**Theorem 3.26.** If \( \tau_1 \) is coarser than \( \tau_2 \), then every \( \tau_1\tau_2\text{-}\delta g \)-open set is a \( \tau_1\tau_2\text{-}g \)-open set.

**Proof.** Let \( A \) be \( \tau_1\tau_2\text{-}\delta g \)-open set. Then, \( F_i \subseteq \tau_i\text{-int}(A) \) whenever \( F_i \subseteq A \) and \( F_i \) is \( \tau_i \)-closed for each \( i = 1, 2 \). Since every closed set is semi-closed, \( F_i \) is \( \tau_i \)-semi closed. Therefore, \( F_i \subseteq \tau_i\text{-int}(A) \), whenever \( F_i \subseteq A \), \( F_i \) is \( \tau_i \)-semi closed. Since \( \tau_1 \) is coarser than \( \tau_2 \), \( F_i \subseteq \tau_2\text{-int}(A) \), whenever \( F_i \subseteq A \), \( F_i \) is \( \tau_1 \)-semi closed. Hence \( A \) is a \( \tau_1\tau_2\text{-}g \)-open set.

**Theorem 3.27.** If \( \tau_1 \) is coarser than \( \tau_2 \), then every \( \tau_1\tau_2\text{-}\delta g \)-open set is a \( \tau_1\tau_2\text{-}g \)-open set.

**Proof.** Let \( A \) be \( \tau_1\tau_2\text{-}\delta g \)-open set. Then, \( F_i \subseteq \tau_i\text{-int}(A) \) whenever \( F_i \subseteq A \) and \( F_i \) is \( \tau_i \)-closed for each \( i = 1, 2 \). Therefore, \( F_i \subseteq \tau_i\text{-int}(A) \), whenever \( F_i \subseteq A \), \( F_i \) is \( \tau_1 \)-closed. Since \( \tau_1 \) is coarser than \( \tau_2 \), \( F_i \subseteq \tau_2\text{-int}(A) \), whenever \( F_i \subseteq A \), \( F_i \) is \( \tau_1 \)-closed. Hence \( A \) is a \( \tau_1\tau_2\text{-}g \)-open set.

**Theorem 3.28.** If \( \tau_1 \) is coarser than \( \tau_2 \), then every \( \tau_1\tau_2\text{-}\delta g \)-open set is a \( \tau_1\tau_2\text{-}g \)-open set.

**Proof.** Let \( A \) be \( \tau_1\tau_2\text{-}\delta g \)-open set. Then, \( F_i \subseteq \tau_i\text{-int}(A) \) whenever \( F_i \subseteq A \) and \( F_i \) is \( \tau_i \)-closed for each \( i = 1, 2 \). Therefore, \( F_i \subseteq \tau_1\alpha \text{int}(A) \), whenever \( F_i \subseteq A \). Since \( \tau_1 \) is coarser than \( \tau_2 \), \( F_i \subseteq \tau_2\alpha \text{int}(A) \), whenever \( F_i \subseteq A \), \( F_i \) is \( \tau_1 \)-closed. Hence \( A \) is a \( \tau_1\tau_2\text{-}g \)-open set.

**Theorem 3.29.** If \( \tau_1 \) is coarser than \( \tau_2 \), then every \( \tau_1\tau_2\text{-}\delta g \)-open set is a \( \tau_1\tau_2\text{-}g \)-open set.

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Proof. Let \(A\) be \(\tau_1\tau_2-\bar{g}\)-open set. Then, \(F_i \subseteq \tau_i\)-int(\(A\)) whenever \(F_i \subseteq A\) and \(F_i\) is \(\tau_i\)-closed for each \(i = 1, 2\). Therefore, \(F_i \subseteq \tau_1\)-int(\(A\)) whenever \(F_i \subseteq A\), \(F_i\) is \(\tau_1\)-closed as every closed set is \(\alpha\)-closed. Since \(\tau_1\) is coarser than \(\tau_2\), \(\tau_2\)-\(\bar{F}_1\) \(\subseteq \alpha\)-int(\(A\)), whenever \(F_1 \subseteq A\), \(F_1\) is \(\tau_1\)-\(\alpha\)-closed. Hence \(A\) is a \(\tau_1\tau_2\)-\(\alpha\)-open set.

\(\square\)

**Theorem 3.30.** If \(\tau_1\) is coarser than \(\tau_2\), then every \(\tau_1\tau_2\)-\(\bar{g}\)-open set is a \(\tau_1\tau_2\)-\(\bar{g}\)-open set.

**Proof.** Let \(A\) be \(\tau_1\tau_2\)-\(\bar{g}\)-open set. Then, \(F_1 \subseteq \tau_1\)-int(\(A\)), \(F_1\) is \(\tau_1\)-closed. Since every closed set is semi closed, \(F_1 \subseteq \tau_1\)-int(\(A\)), whenever \(F_1 \subseteq A\), \(F_1\) is \(\tau_1\)-semi closed. Since \(\tau_1\) is coarser than \(\tau_2\), \(F_1 \subseteq \tau_2\)-int(\(A\)), whenever \(F_1 \subseteq A\), \(F_1\) is \(\tau_1\)-semi closed. Hence \(A\) is a \(\tau_1\tau_2\)-\(\bar{g}\)-open set.

\(\square\)

**Definition 3.31.** A function \(f\) from spaces \((X, \tau_1, \tau_2)\) into \((Y, \sigma_1, \sigma_2)\) is called \(\tau_1\tau_2 - \bar{g}\)-continuous if \(f^{-1}(V)\) is \(\tau_1\tau_2 - \bar{g}\)-open set in \(X\) for each \(\sigma_1\)-open set \(V\) in \(Y\).

**Example 3.32.** Let \(X = \{a, b, c\} = Y; \tau_1 = \{\phi, \{a, b\}, X\}; \tau_2 = \{\phi, \{b\}, \{a, c\}, X\}; \sigma_1 = \{\phi, Y\}\) and \(\sigma_2 = \{\phi, \{b\}, Y\}\). Then \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) defined by \(f(a) = a\) is \(\tau_1\tau_2 - \bar{g}\)-continuous mapping.

**Theorem 3.33.** If \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is \(\tau_1\tau_2 - \bar{g}\)-continuous and \(g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \rho_1, \rho_2)\) is continuous, then \(g \circ f\) is \(\tau_1\tau_2 - \bar{g}\)-continuous.

**Proof.** Let \(A\) be \(\rho_1\)-open set in \(Z\). Since \(g\) is continuous, \(g^{-1}(A)\) is \(\sigma_1\)-open in \(Y\). Since \(f\) is \(\tau_1\tau_2\)-\(\bar{g}\)-continuous, \(f^{-1}(g^{-1}(A))\) is \(\tau_1\tau_2\)-\(\bar{g}\)-open in \(X\). Hence \(g \circ f\) is \(\tau_1\tau_2\)-\(\bar{g}\)-continuous. \(\square\)

### 4 Conclusion

In this paper, \(\tau_1\tau_2\)-\(\bar{g}\)-open sets were introduced in the bitopological spaces and their properties were studied. Further, their properties were compared with some of the corresponding generalized open sets in the general topological spaces and bitopological spaces.

### Competing Interests

Authors have declared that no competing interests exist.

### References


[22] Indira T. $\tau_1\tau_2$-$\tilde{g}$-closed sets in Bitopological spaces. Annals of Pure and Applied mathematics. 2014;7(2):27-34.


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