



Egyptian Mathematics Disclosure

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Some fundamental mathematical researches have been carried out about mathematical certainties based on ancient Egyptian mathematical sources and their problems following ancient Egyptian Wisdom set of knowledge building the new scientific paradigm following the rediscovery of the true value of PI and following the new approach of Global Dimensional Mathematics [1].

Some fundamental mathematical researches on the foundations of Egyptian mathematics covering the mathematical problem of The *Akhmin wooden tablets* [2], the tenth and the fourteenth problem of The *Moscow Mathematical Papyrus* [3] as well as the forty-first and fiftieth problem from The *Rhind Mathematical Papyrus* [3] have been carried out, without forgotten, the resolution of the fundamental question of the quadrature of the circle which is now effective.

In the disclosure of Egyptian mathematics, the new approach to fundamental mathematical notions is established, adding the cornerstone to building the core of the new approach to Egyptian mathematics, mathematics and science in general.

The Egyptian mathematics disclosure solves, following the Egyptian approach to mathematics and following ancient Egyptian Wisdom set of knowledge, unsolved ancient Egyptian mathematical problems, such as finding the complete solution and decoding the glyph of the eye of Horus, as well as the problem of the truncated pyramid which has found a solution like the half basket problem found one. The question of the quadrature of the circle shatters the mathematical conceptions with all the consequences that we can only

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begin to understand.

The Egyptian mathematics disclosure forms the basis for building the new scientific approach based on ancestral Egyptian mathematical problems, the true rediscovered value of PI and the new original Global Dimensional Mathematics opening up a still unknown perspective on the world of science in general.

Keywords: Egyptian mathematics; quadrature of the circle; eye of horus; PI; Paradigm shift.

1 Introduction

Some researches on the foundations of Egyptian mathematics covering the mathematical problem of The *Akhmin wooden tablets* [2], the tenth and the fourteenth problem of The *Moscow Mathematical Papyrus* [3] as well as the forty-first and fiftieth problem from The *Rhind Mathematical Papyrus* [3] have been carried out, without forgotten, the resolution of the fundamental question of the quadrature of the circle which is now effective. Here we reconstruct the fundamental mathematical certainties based on ancient Egyptian mathematical sources and their problems following ancient Egyptian Wisdom set of knowledge building the new scientific paradigm following the rediscovery of the true value of PI and following the new approach of Global Dimensional Mathematics [1]. The Egyptian mathematics disclosure forms the basis for building the new scientific approach based on ancestral Egyptian mathematical problems, the true rediscovered value of PI and the new original Global Dimensional Mathematics opening up a still unknown perspective on the world of science in general.

2 Methodology

Our current understanding of ancient Egyptian mathematics is imputed to the paucity of available sources. The existing sources include the following texts dated to the Middle Kingdom and Second Intermediate Period:

- The Moscow Mathematical Papyrus [3]
- The Egyptian Mathematical Leather Roll [3]
- The Lahun Mathematical Papyri [3]
- The Berlin Papyrus 6619, written around 1800 BC
- The Akhmim Wooden Tablet [2]
- The Reisner Papyrus, dated to the early Twelfth dynasty of Egypt and found in Nag el-Deir, the ancient town of Thinis [3]
- The Rhind Mathematical Papyrus (RMP), dated from the Second Intermediate Period (c. 1650 BC), but its author, Ahmes, identifies it as a copy of a now lost Middle Kingdom papyrus. The RMP is the largest mathematical text. [3]

Some problems in both, the Moscow Mathematical Papyrus (MMP) and in the Rhind Mathematical Papyrus (RMP) are geometric problems proving by the used examples that the Ancient Egyptians knew how to compute areas and the volumes of several geometric shapes as cylinders and pyramids.

- **Area:**
 - **Triangles:** The scribes record problems about the computation of the area of a triangle (RMP and MMP). [3]
 - **Rectangles:** Some problem 50 in the RMP and the MMP are based on the calculation of the area of a rectangular plot of land. [3] In the Lahun Mathematical Papyri in London appears a similar problem [4,5]
 - **Circles:** Compares the area of a circle (approximated by an octagon) and its circumscribing square it is the Problem 48 of the RMP, this problem's result is used in another problem, where the scribe finds the area of a round field of diameter 9 khet in problem 50 [3]
 - **Hemisphere:** finds the area of a hemisphere in Problem 10 in the MMP [3]

- **Volumes:**

- **Cylindrical granaries:** In section IV.3 of the Lahun Mathematical Papyri the volume of a cylindrical granary is computed using the same procedure as in the RMP 43. The RMP 41 and 42 problem are equally problem about the computation of the volume of a cylindrical granary, while a pillar or a cone instead of a pyramid seems to be concerned in the problem 60 RMP. It is rather small and steep, with a seked (reciprocal of slope) of four palms (per cubit) [3].
- **Rectangular granaries:** In the Moscow Mathematical Papyrus (problem 14) and in the Rhind Mathematical Papyrus (numbers 44, 45, 46) Several problems concern the computation of the volume of a rectangular granary [3,4]
- **Truncated pyramid (frustum):** In the MMP the problem 14 is about the computation of the volume of a truncated pyramid [3].

In the ancient Egyptian mathematics system, an interesting feature is the use of unit fractions [6]. Some special notation for fractions such as $1/2$, $1/3$ and $2/3$ and in some texts for $3/4$ were used by the Egyptians but other fractions were all written as unit fractions of the form $1/n$ or sums of such unit fractions. Such tables were used by the Scribes to help them work with these fractions. The Egyptian Mathematical Leather Roll for instance is a table of unit fractions which are expressed as sums of other unit fractions. The Rhind Mathematical Papyrus and some of the other texts contain $2/n$ tables. The scribes were helped by these tables and allowed them to rewrite any fraction of the form $1/n$ as a sum of unit fractions [7].

Some knowledge of geometrical progression is found with the use of the Horus eye fractions moreover some arithmetical knowledge as progressions are also evident from the mathematical sources [8].

The first civilization to develop and solve second-degree (quadratic) equations were the ancient Egyptians. This information is found in the Berlin Papyrus fragment. Moreover in Rhind Mathematical Papyrus, first-degree algebraic equations are solved by the Egyptians [9].

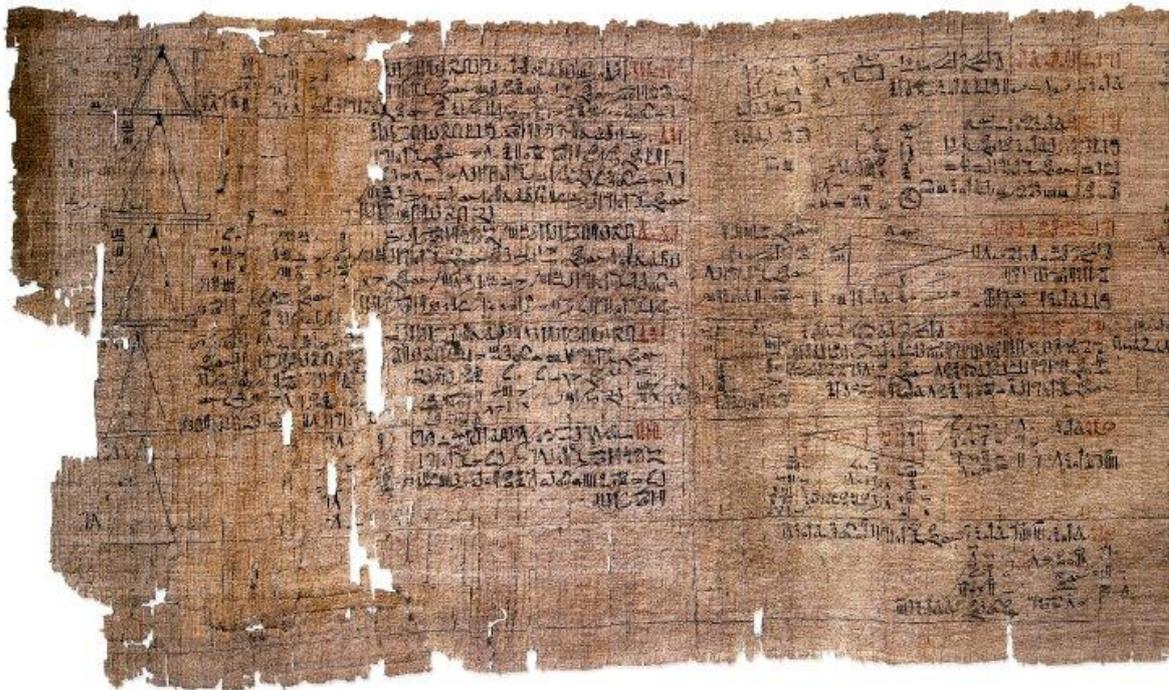


Fig. 1. A portion of Ahmes papyri. Written by a scribe named Ahmose in 1550 BCE. Problems in Algebra, geometry and other areas in mathematics are listed and covered in the Papyrus bought by Alexander Henry Rhind into in modern times

2.1 Glyph of eye of horus decoded

In the imagery of the ancient Egyptians, the Eye of Horus (wꜥḏt) is an important symbol. It represents a human eye sharing characteristics with the eye of a hawk.

Following a frequent pun between the word for eye and the word action in Egyptian, the Eye of Horus also referred to the economic output of Pharaonic Egypt under the reign of each Pharaoh. In this perspective of interpretation, it has been proposed that the representation of the Udjat eye could be broken down into different units of grain measurement.



Fig. 2. Interpretation of the value of the oudjat eye

Egyptians might have perceived these unit fractions as "units" to be added as we similarly build up integers by summing the decimal units system.

This proved that the Egyptian fractions were indeed foundation of many Egyptian mathematical development and civilization during that period of time.

KEY of EYE OF HORUS

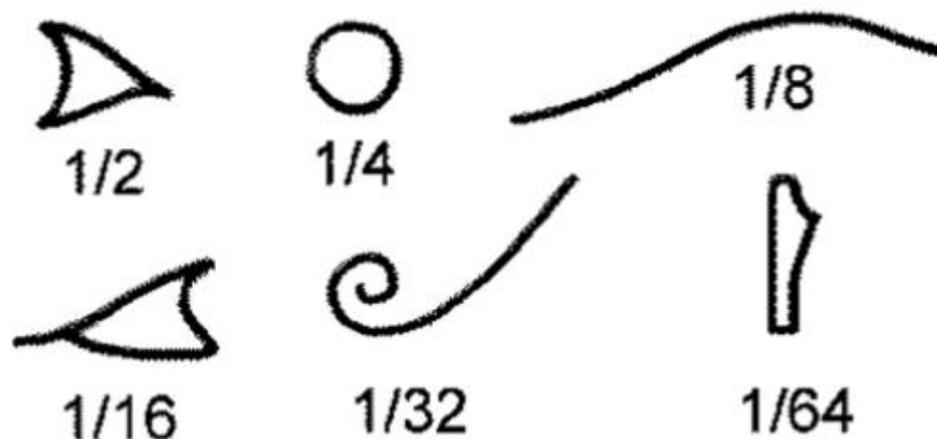


Fig. 3. Eye of Horus: Fraction composition and numerical value

Dating to 2000 BC, near the beginning of the Egyptian Middle Kingdom, housed in the Egypt Museum in Cairo, a document often called the Cairo wooden tablet, The Akhmim wooden tablet.

The document was believed for nearly 100 years to define a unit known as a "ro" as being equal to 1/320th of a hekat, when in fact an exact computation was taking place that requires no additional units of the hekat.

The "ro" operation was simply as used to exactly complete the remainder fraction component of the following divisions:

Akhmim Wooden Tablet

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fraction	identity
$64/3 = 21 + 1/3$	$5(3 \text{ ro}) = 5/960$
$64/7 = 9 + 1/7$	$5(7 \text{ ro}) = 5/2240$
$64/10 = 6 + 2/5$	$10(10 \text{ ro}) = 1/320$
$64/11 = 5 + 9/11$	$45(11 \text{ ro}) = 45/3520$
$64/13 = 4 + 12/13$	$60(13 \text{ ro}) = 60/4160$

Fig. 4. 05-02-10 Akhmim Wooden Tablet -- from Wolfram MathWorld

The Egyptians used two numbering systems:

- the hieroglyphic system (on monuments), based on 10, non-positional
- the hieratic system (on papyri), based on 10, which avoids the repetition of certain signs.

They only wrote integers and fractions of the unit. Moreover, the fraction 2/3 has a particular importance and a particular symbol.

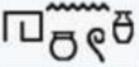
Names		
English	Egyptian	
Ro		r
Dja		dja
Jar Hinw		hnw
Barrel Heqat Hekat		hqt
Double Barrel Double Heqat Double Hekat		hqty
Quadruple Heqat (MK) ^[44] Oipe ^[45] (NK) ^[44]		hqt-fdw jpt ^[19] ipt ^[44]
Sack Khar		khar
Deny Cubic cubit		deny

Fig. 5. Egyptian « ro » representation

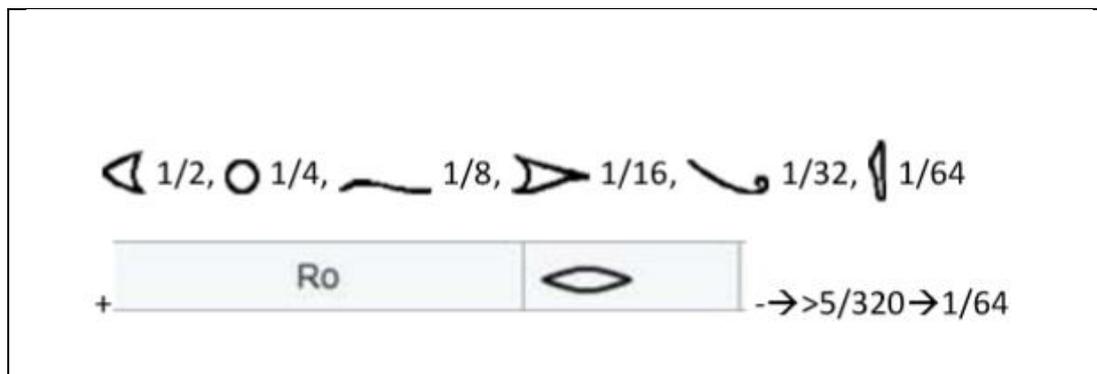


Fig. 6. Fraction value and representation of Complete Eye of Horus

Multiplications are made by additions from multiplications and divisions by 2.

An additional decimal number system was used by the Egyptians, they did not know zero, fractions were all written as unit fractions of the form $1/n$ or sums of such unit fractions, fractions such as $1/2$, $1/3$ and $2/3$ and in some texts for $3/4$ were known. Tables were used to help scribes to work with these fractions. Egyptian fractions were used in Greece and during the Middle Ages and are connected with some modern problems in number theory.

$1/2, 1/4, 1/8, 1/16, 1/32$

so
 $1/2, 1/2^2, (1/2^2)*(1/2), 1/4^2, (1/4^2)*(1/2)$

But this kind of approach is far beyond the scope of this article

2.2 Moscow Mathematical Papyrus Problem 14: Volume of frustum

A truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top:

You are to square the 4; result 16.

You are to double 4; result 8.

You are to square this 2; result 4.

You are to add the 16 and the 8 and the 4; result 28.

You are to take 1/3 of 6; result 2.

You are to take 28 twice; result 56.

See, it is of 56. You will find [it] right"

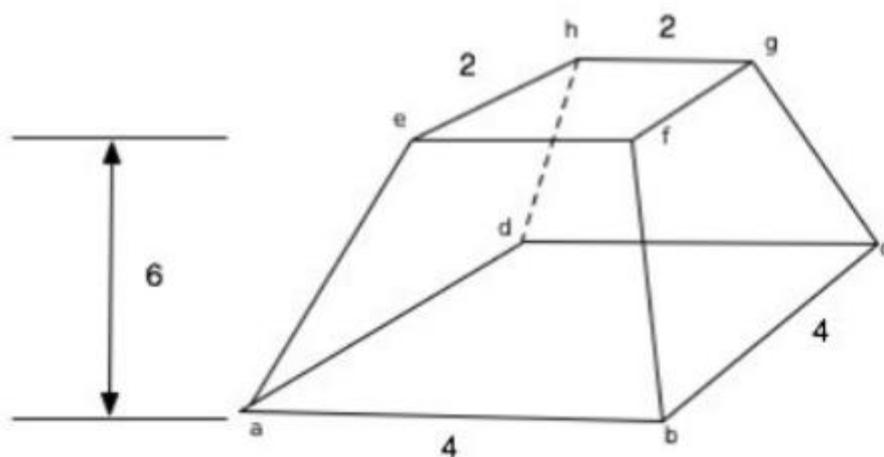


Fig. 7. Drawn of the issue 14 of Moscow Mathematical Papyrus

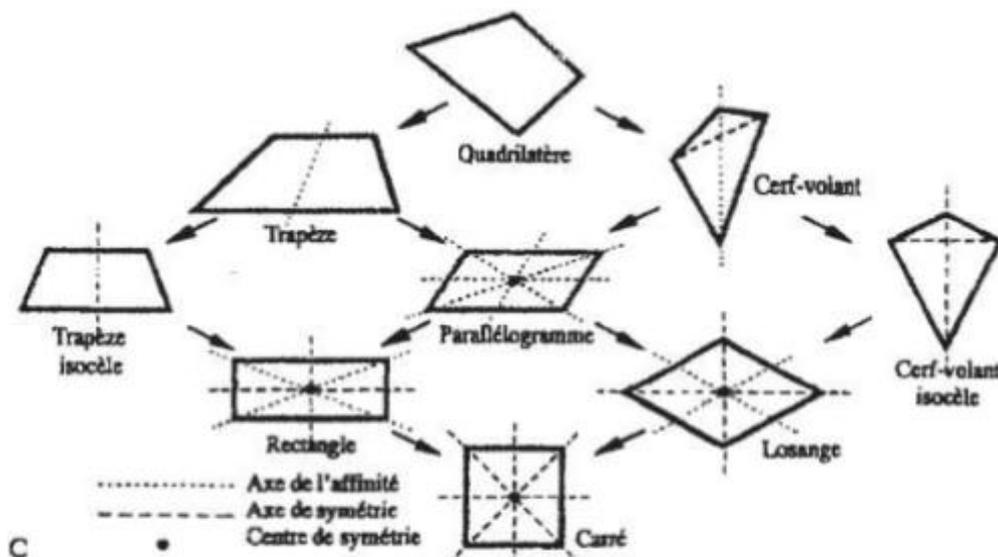


Fig. 8. Classification of Quadrilateral

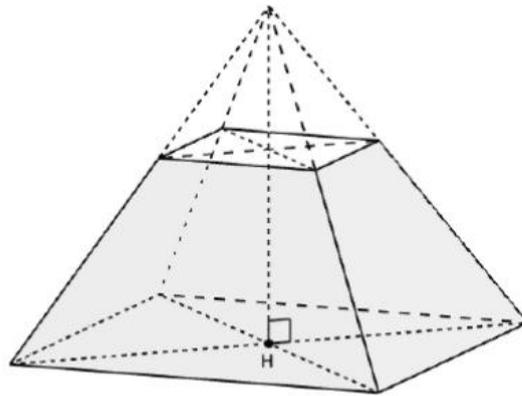


Fig. 9. Truncated Pyramid

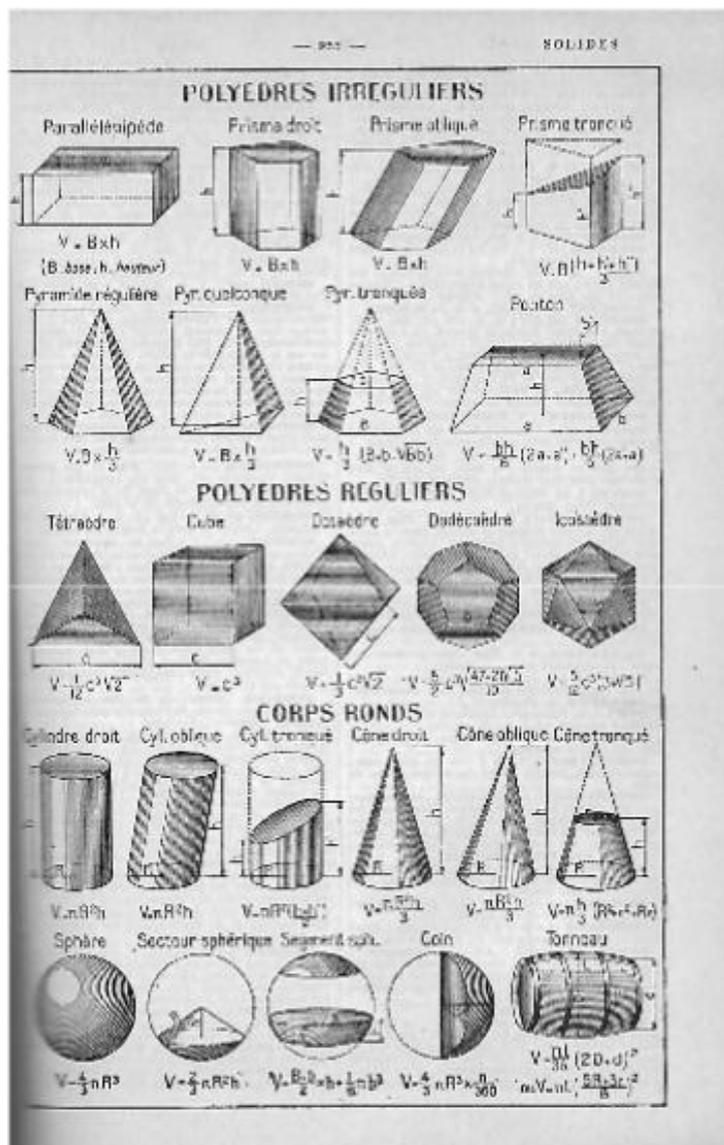


Fig. 10. Le « Nouveau Petit Larousse Illustré » 1952

$$V = \frac{h \times (B + b + \sqrt{Bb})}{3}$$

B:area big base

b:area little base

h=height

Fig. 11. Volume of frustum following : Le « Nouveau Petit Larousse Illustré » 1952

- 1 4—area top base
- 2 8—2*4 area rectangle
- 4 16---area bottom base

28

*Height/3

2*28=56

Or

$$\frac{h}{3} * (B + b + Bb)$$

Fig. 12. Egyptian formula for square frustum

With

B : Area of big base

b : Area of little base

Bd : Area of third base

This result can be put in relation with the concept of dimension and space developed in Global Dimensional Mathematics.

Indeed 1 Dimension has 3 directions and a complete space has 3 dimensions so 3*3 directions spatial + 2 direction no spatial.

So, we divide 3 Dimension (9 directions) per 3 to have for 3 directions corresponding to our space 3 Directions is 1 Dimension [1]

2.3 Quadrature of the Circle in Ancient Egypt

2.3.1 Rhind papyrus Problem 50: Area of a Field

Example of a round field of diameter 9 khet. What is its area?

Take away 1/9 of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore, it contains 64 *setat* of land [10].

A circular field has diameter 9 khet.

What is its area.

1 khet = 100 meh

9 khet = 900 meh

The written solution says, subtract 1/9 of of the diameter which leaves 8 khet

$8 \times 100 \text{ meh} = 800 \text{ meh}$

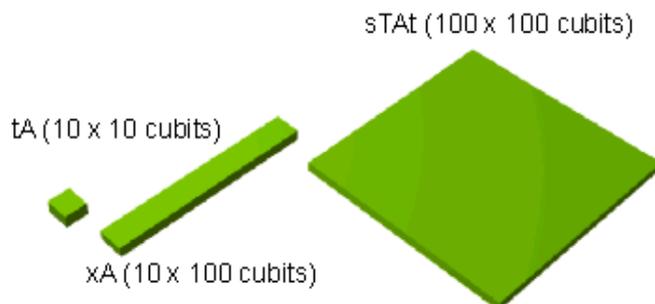
The area is 800 multiplied by 800, 640000 or 64 setat

The setat was the basic unit of land measure and may originally have varied in size across Egypt's nomes.

Measuring area in Ancient Egypt

The recorded units of measurement in Old Kingdom written sources are:

- **land-unit** (Egyptian tA) = 10×10 cubits (about 27.65 square metres)
- **thousand** (Egyptian xA) = 10×100 cubits (about 275.65 square metres)
- **setjat** (Egyptian sTat) = 100×100 cubits (about 2756.5 square metres)



The smaller units are:

- **shoulder** (Egyptian rmn) = half tA
- **account unit** (Egyptian Hsb) = half rmn
- **sA** (Egyptian sA) = half Hsb

New Kingdom (about 1550-1069 BC) sources use different vocabulary:

- **mH-tA** = 100×100 cubits (about 27.5 square metres)
- **sTAt** = 1000×1000 cubits (about 2756.5 square metres) **the aroura** of Greek sources
- **xA-tA** = 10 sTAt (about 2.75 hectares)

Fig. 13. Measuring area in ancient Egypt

Later, it was equal to one square khet, where a khet measured 100 cubits.

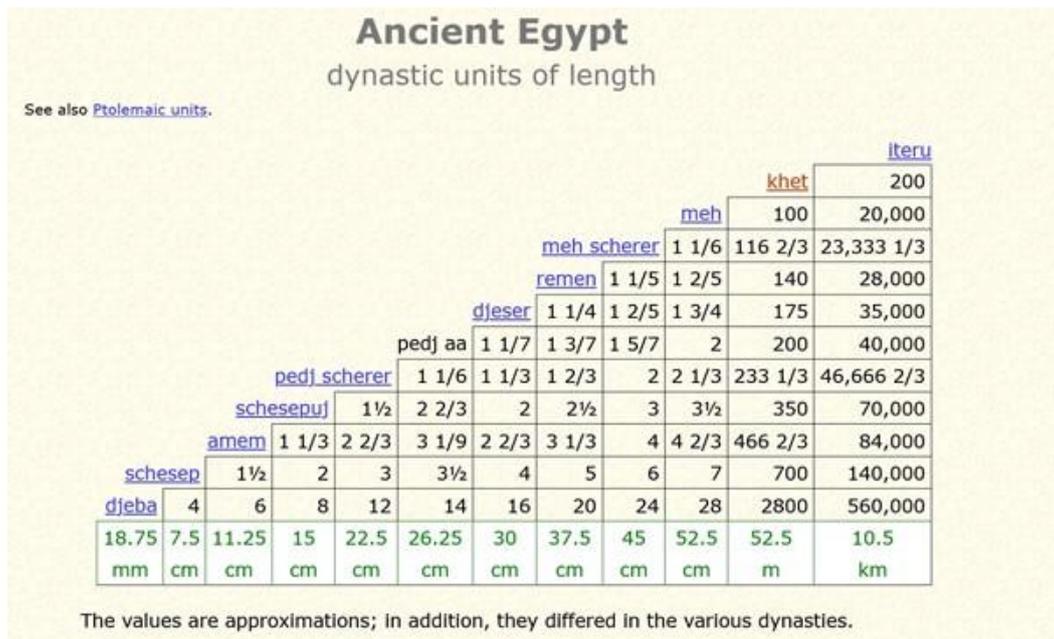


Fig. 14. Ancient Egypt Dynastic units of Length



Fig. 15. Ancient Egypt Ptolemaic units of Length

mahi

In ancient Egypt, *Dynasty IV to Dynasty* , a unit of length, the cubit, 20.51 inches to 20.76 inches.

Fig. 16. Ancient Egypt Dynastic evolution mahi, meh units of Length

Take 1/9 of 9 (since) the basket is half an egg-shell.

Modus operandi:

Basket:: area half circle

So area of complete circle (9-(1/9))²

So area of basket == ((9-(1/9))²)/2:: half circle

Take away 1/9 of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64000. Therefore 64 setat/2
==32

The setat was the basic unit of land measure and may originally have varied in size across Egypt's nomes.

2.3.2 Rhind Mathematical Papyrus issue : 41th

Calculation example of a round granary of [diameter] 9 [and height] 10.

Take away 1/9 of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64000. Therefore 64 setat

You multiply 64 par 10. It comes 640.

$$V = h \cdot (d - d/9)^2$$

2.3.3 Introduction

One of the problems given as solved by the Rhind [3] papyrus, written around 1650 BC. AD, gives the square of side 8 as the same area as a circle of diameter 9, which amounts to taking for the number π the approximate value $3 + 1/9 + 1/27 + 1/81 = 3.16049382716\dots$

The Rhind papyrus, or handbook of the scribe Ahmes, provides the following statement: "Rule for calculating a round field of 9 poles." What is its capacity? Take 1/9, it's 1. [Subtract from 9], remainder 8. Multiply the number 8 eight times, that gives 64. Its capacity is 64" [11].

The decimals of Pi have been the prey of scholars for almost 4,000 years.

One of Pi's oldest approximations is found on the famous Rhind papyrus copied by the scribe Ahmes.

Let us quote from him:

"The area of the circle of diameter 9 cubits is that of the square of side 8 cubits."

2.3.4 Rhind Mathematical Papyrus issue : 48

Problem 48 of the RMP compares the area of a circle and its circumscribing square.

Example of a circle of diameter 9.

Take away 1/9 of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64000. Therefore it 64 setat.

Circle = 64

Square of 9 → 9²=81

2.3.5 Proof

We see here the calculation of the area of a circle with the Egyptian method and Greek method and result comparison of the error considered during those calculation.

For a circle of 90 in diameter:

- $(90 - (90/9))^2 = 6400 \rightarrow$ Egyptian Area
- $(45)^2 * \pi = 6361.72512352 \rightarrow$ Greeks Area

Variation absolute:

$$6400 - 6361.72512352 = 38.2748764807$$

Variation relative:

$$38.2748764807 / 6400 = 0.00598044945$$

For a 70-diameter circle:

- $(70 - (70/9))^2 = 3871.60493827 \rightarrow$ Egyptian Area
- $(35)^2 * \pi = 3848.45100065 \rightarrow$ Greeks Area

Variation absolute:

$$3871.60493827 - 3848.45100065 = 23.15393762$$

Variation relative:

$$23.15393762 / 3871.60493827 = 0.00598044944$$

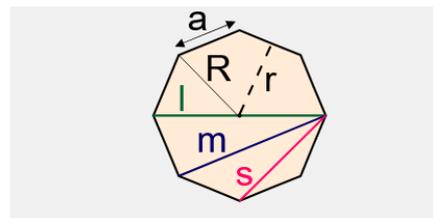
+ error from PI:

$$3871.60493827 - (0.00598044944 * 3871.60493827) = 3848.45100069$$

The approximation of the Greek value PI in regard to the calculation of the area of the circle corresponds to the calculation of the area of a circle from the Egyptian formulas with an relative error

2.3.6 Circle approximation

Problem 48 of the RMP compares the area of a circle (approximated by an octagon) and its circumscribing square.



Side length (a)	9 cm ▾
Perimeter	72 cm ▾
Area	391.1 cm² ▾
Longest diagonal (l)	23.52 cm ▾
Medium diagonal (m)	21.73 cm ▾
Shortest diagonal (s)	16.63 cm ▾
Circumcircle radius (R)	11.76 cm ▾
Incircle radius (apothem) (r)	10.864 cm ▾

Fig. 17. Octagon Calculator, By Álvaro Díez and Bogna Szyk

<https://www.omnicalculator.com/math/octagon>

Trisect each side. Remove the corner triangles. The resulting octagonal figure approximates the circle. The area of the octagonal figure is:

$$9^2 - 4 \cdot \frac{1}{2} (3)(3) = 63$$

Next we approximate 63 to be 64 and note that

Thus the number $4 \left(\frac{8}{9}\right)^2 = 3.16049\dots$ (plays the role of $\pi = 3.14159\dots$)

That this octagonal figure, whose area is easily calculated, so accurately approximates the area of the circle. It is not simple to Obtain a better approximation to the area using finer divisions of a square and a similar argument.

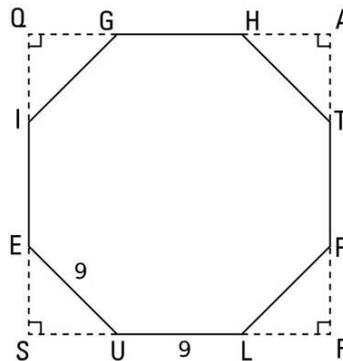
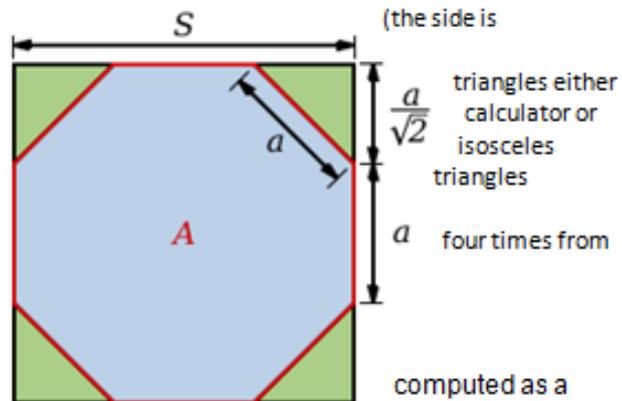


Fig. 18. The four corners (like triangle *SUE* in the figure) that you cut off the square to turn it into an octagon are 45°- 45°- 90° triangles. So calculate the area of the square and then subtract the four corner triangles

if you imagine an octagon shape inside of a square you can see that the difference is only four right triangles. "How to find the area of a regular octagon with this information?"

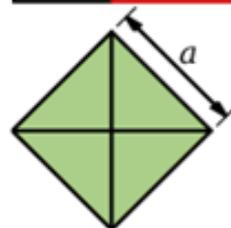
1. Calculate the area of the square $2 \cdot \text{apothem}$,
2. Calculate the sides of the right by using the 45 45 90 triangle the fact that they are right triangles, and use the special right calculator.
3. Subtract the area of a right triangle the area of the square,



The area of a regular octagon can be truncated square.

The area can also be expressed as

Another simple fomula for the area is



$$A = S^2 - a^2$$

$$A = 2aS.$$

Fig. 19. Area of a regular octagon (accurately approximates the area of the circle) can be computed as a truncated square

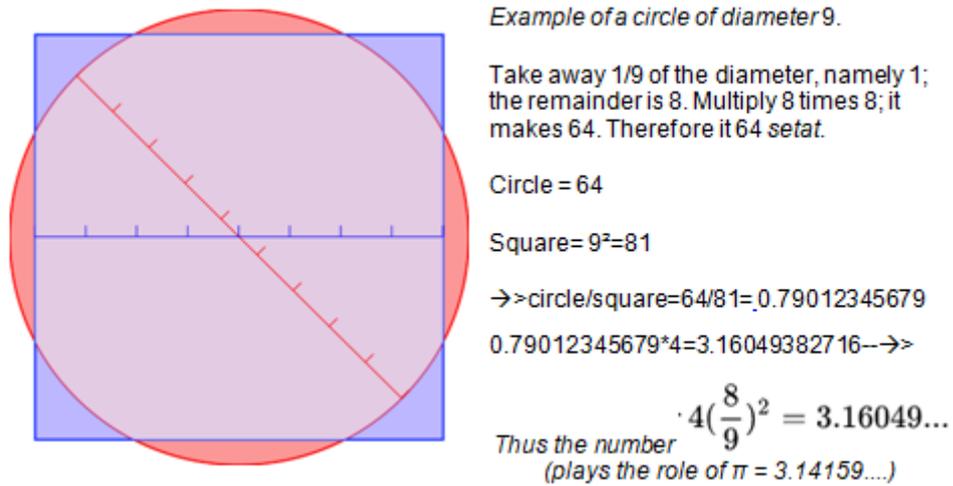


Fig. 20. Egyptian construction of Problem 48 from the Rhind papyrus

3 Discussion

The rediscovery of the original value of PI following the ancient Egyptian formulae and some problems resolutions about ancient historical mathematics issues leads to news mathematical truth in regard to the Greek approximation of PI. We find with the Egyptian formulae of PI given in several different Egyptian mathematical references papyri the real value of PI without implicates the call to a transcendental number as in the inaccurate Greek approximation of PI.

This rediscovery of PI is consistent, relevant with the set of knowledge presented in the multiple Egyptian mathematical references papyri based on the ancient Egyptian wisdom.

4 Conclusions

The Egyptian mathematics disclosure leads with the help of the ancient mathematical references Egyptian papyri and some historical issues resolution to new conception of mathematical truth about some calculus, mathematic concepts, and even, symbolic one with the decoded glyph of EYE of HORUS leading to a total change of paradigm...

Moreover, the rediscovery of real truth Egyptian value of Pi consistent, relevant and coherent with ancient the Egyptian wisdom knowledge breaks the inaccurate Greek approximation of Pi based on transcendental number use.

The Egyptian mathematics disclosure based on ancient Egyptian mathematics from several Egyptian mathematical references papyri following ancient Egyptian Wisdom set of knowledge throws the basis for building the new scientific paradigm of the twenty-first century and beyond, based on Global Dimensional mathematics opening up an unknown perspective regarding the world of science in general.

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Disclaimer

The author declares no conflicts of interest regarding the publication of this paper.

Competing Interests

Author has declared that no competing interests exist.

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